Circular Motion

Angular speed

$$v = r\omega$$

An object is travelling on a circle with radius r, and moves between the points A and B with speed vms^{-1}.

We should know that the length of l between A and B can be expressed by the equation $l = r\theta$, where r is the length of the radius and θ is the angle measured in radians.

$$v = \frac{\Delta s}{\Delta t}$$

which as discussed

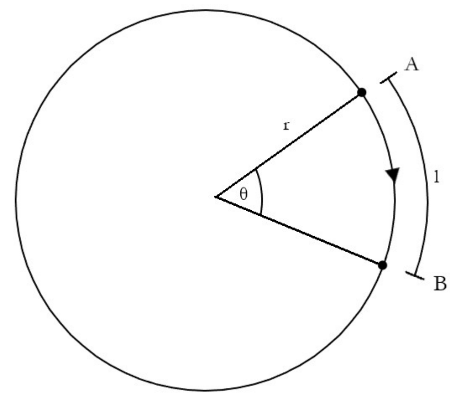
$$v = \frac{\Delta r\theta}{\Delta t}$$

But since the radius of the circle remains constant

$$v = r\frac{\Delta \theta}{\Delta t}$$

And so we have our equation of angular speed, where $\omega = \frac{\Delta\theta}{\Delta t}$

$$v = r\omega$$



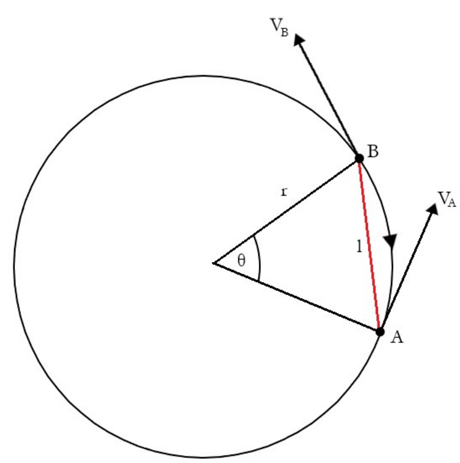
For example, if a car travels around a roundabout of radius $2m$ with speed $5ms^{-1}$ then its angular speed will be $\omega = \frac{5}{2} = 2.5\ rads^{-1}$.
ms^{-1} is another way of writing m/s and $rads^{-1}$ is another way of writing rad/s. These units make sense as speed is distance over time, and angular speed is angle over time.

Angular acceleration

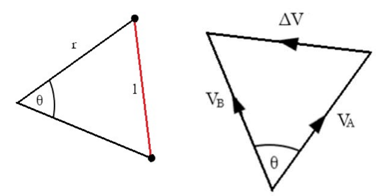
$$a = r\omega^2$$

Even if the speed of an object stays the same, its velocity will change as the direction it is moving in is constantly changing. If velocity changes, this means the object must always have an acceleration.

Consider an object travelling on a circle, again its speed is v. Its velocity at A is v_A and at B it is v_B. Therefore its change in velocity can be found by the equation $\Delta v = v_B - v_A$
When θ is small, we can say that the distance between A and B (l) can be approximately expressed as a straight line from A to B, as there is little curvature of the arc.



We then have two triangles we can draw, shown below. Clearly, these



triangles are similar[1] as they both have angle θ and both are isosceles. For the far left triangle, two sides are of length r and for the nearer triangle two sides v_A and v_B are of the same length as they are of the same magnitude. This means the ratio between l and r is the same as that between Δv and v_A, so

$$\frac{l}{r} = \frac{\Delta v}{v}$$

We can use the fact that $v = \frac{\Delta s}{\Delta t}$, in this case $l = \Delta s = v\Delta t$

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

If we multiply both sides by $\frac{v}{\Delta t}$

$$\frac{v^2}{r} = \frac{\Delta v}{\Delta t}$$

Since $a = \frac{\Delta v}{\Delta t}$

$$a = \frac{v^2}{r}$$

and $v = r\omega$

$$a = \frac{\omega^2 r^2}{r}$$

so

$$a = r\omega^2$$

Revisiting the car, which has $\omega = 2.5rads^{-1}$ and $r = 2m$, its acceleration can therefore be found by $a = 2.5^2 \times 2 = 12.5ms^{-2}$

<u>See also</u>
- Circles

<u>References</u>
Alldridge, T. et al. (2015). *A-level Physics*. Newcastle upon Tyne: Coordination Group Publications. pp.96-97.

[1] Meaning they have the same angles and are of the same proportions